1. **(A)** The number M is equal to

$$\frac{1}{9}(9+99+999+\ldots+999{,}999{,}999) = 1+11+111+\ldots+111{,}111{,}111 = 123{,}456{,}789.$$

The number M does not contain the digit 0.

2. **(D)** Since

$$\begin{aligned}(3x-2)(4x+1)-(3x-2)4x+1 &= (3x-2)(4x+1-4x)+1\\ &= (3x-2)\cdot 1+1 = 3x-1,\end{aligned}$$

when $x = 4$ we have the value $3\cdot 4 - 1 = 11$.

3. **(B)** If $n \geq 4$, then

$$n^2-3n+2 = (n-1)(n-2)$$

is the product of two integers greater than 1, and thus is not prime. For $n = 1$, 2, and 3 we have, respectively,

$$(1-1)(1-2) = 0, \quad (2-1)(2-2) = 0, \quad \text{and} \quad (3-1)(3-2) = 2.$$

Therefore, n^2-3n+2 is prime only when $n = 3$.

4. **(E)** The number $\frac{1}{2}+\frac{1}{3}+\frac{1}{7}+\frac{1}{n}$ is greater than 0 and less than $\frac{1}{2}+\frac{1}{3}+\frac{1}{7}+\frac{1}{1} < 2$. Hence,

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{7}+\frac{1}{n} = \frac{41}{42}+\frac{1}{n}$$

is an integer precisely when it is equal to 1. This implies that $n = 42$, so the answer is (E).

5. **(D)** A pentagon can be partitioned into three triangles, so the sum of the degree measures of the angles of the pentagon is

$$v+w+x+y+z = 540.$$

The arithmetic sequence can be expressed as $x-2d$, $x-d$, x, $x+d$, and $x+2d$, where d is the common difference, so

$$(x-2d)+(x-d)+x+(x+d)+(x+2d) = 5x = 540.$$

Thus, $x = 108$.

6. **(C)** The given conditions imply that

$$x^2 + ax + b = (x - a)(x - b) = x^2 - (a + b)x + ab,$$

so

$$a + b = -a \quad \text{and} \quad ab = b.$$

Since $b \neq 0$, the second equation implies that $a = 1$. The first equation gives $b = -2$, so $(a, b) = (1, -2)$.

7. **(B)** Let $n - 1$, n, and $n + 1$ denote the three integers. Then

$$(n - 1)n(n + 1) = 8(3n).$$

Since $n \neq 0$, we have $n^2 - 1 = 24$. It follows that $n^2 = 25$ and $n = 5$. Thus,

$$(n - 1)^2 + n^2 + (n + 1)^2 = 16 + 25 + 36 = 77.$$

8. **(D)** Since July has 31 days, Monday must be one of the last three days of July. Therefore, Thursday must be one of the first three days of August, which also has 31 days. So Thursday must occur five times in August.

9. **(C)** We have $b = a + r$, $c = a + 2r$, and $d = a + 3r$, where r is a positive real number. Also, $b^2 = ad$ yields $(a + r)^2 = a(a + 3r)$, or $r^2 = ar$. It follows that $r = a$ and $d = a + 3a = 4a$. Hence $\dfrac{a}{d} = \dfrac{1}{4}$.

10. **(A)** Each number in the given set is one more than a multiple of 3. Therefore the sum of any three such numbers is itself a multiple of 3. It is easily checked that every multiple of 3 from $1 + 4 + 7 = 12$ through $13 + 16 + 19 = 38$ is obtainable. There are 13 multiples of 3 between 12 and 48 inclusive.

11. **(E)** The numbers $A - B$ and $A + B$ are both odd or both even. However, they are also both prime, so they must both be odd. Therefore, one of A and B is odd and the other even. Because A is a prime between $A - B$ and $A + B$, A must be the odd prime. Therefore, $B = 2$, the only even prime. So $A - 2$, A, and $A + 2$ are consecutive odd primes and thus must be 3, 5, and 7. The sum of the four primes 2, 3, 5, and 7 is the prime number 17.

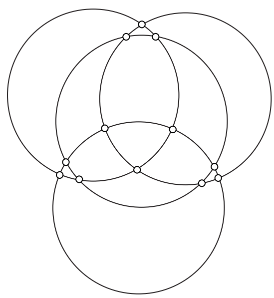
12. **(D)** If $\frac{n}{20-n} = k^2$, for some $k \geq 0$, then $n = \frac{20k^2}{k^2+1}$. Since k^2 and k^2+1 have no common factors and n is an integer, k^2+1 must be a factor of 20. This occurs only when $k = 0, 1, 2,$ or 3. The corresponding values of n are $0, 10, 16,$ and 18.

13. **(B)** Let $n, n+1, \ldots, n+17$ be the 18 consecutive integers. Then the sum is

$$18n + (1 + 2 + \cdots 17) = 18n + \frac{17 \cdot 18}{2} = 9(2n + 17).$$

Since 9 is a perfect square, $2n + 17$ must also be a perfect square. The smallest value of n for which this occurs is $n = 4$, so $9(2n + 17) = 9 \cdot 25 = 225$.

14. **(D)** Each pair of circles has at most two intersection points. There are $\binom{4}{2} = 6$ pairs of circles, so there are at most $6 \times 2 = 12$ points of intersection. The following configuration shows that 12 points of intersection are indeed possible:



15. **(D)** Let a denote the leftmost digit of N and let x denote the three-digit number obtained by removing a. Then $N = 1000a + x = 9x$ and it follows that $1000a = 8x$. Dividing both sides by 8 yields $125a = x$. All the values of a in the range 1 to 7 result in three-digit numbers.

16. **(C)** The product will be a multiple of 3 if and only if at least one of the two rolls is a 3 or a 6. The probability that Juan rolls 3 or 6 is $2/8 = 1/4$. The probability that Juan does not roll 3 or 6, but Amal does is $(3/4)(1/3) = 1/4$. Thus, the probability that the product of the rolls is a multiple of 3 is

$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$

17. **(B)** Let A be the number of square feet in Andy's lawn. Then $A/2$ and $A/3$ are the areas of Beth's lawn and Carlos' lawn, respectively, in square feet. Let R be the rate, in square feet per minute, that Carlos' lawn mower cuts. Then Beth's mower and Andy's mower cut at rates of $2R$ and $3R$ square feet per minute, respectively. Thus,

$$\text{Andy takes } \frac{A}{3R} \text{ minutes to mow his lawn,}$$

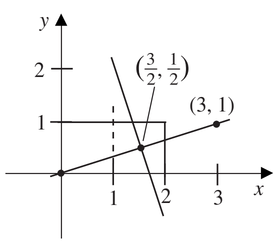
$$\text{Beth takes } \frac{A/2}{2R} = \frac{A}{4R} \text{ minutes to mow hers,}$$

and

$$\text{Carlos takes } \frac{A/3}{R} = \frac{A}{3R} \text{ minutes to mow his.}$$

Since $\dfrac{A}{4R} < \dfrac{A}{3R}$, Beth will finish first.

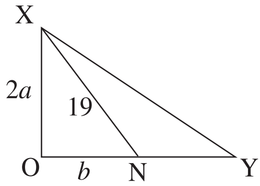
18. **(C)** The area of the rectangular region is 2. Hence the probability that P is closer to $(0,0)$ than it is to $(3,1)$ is half the area of the trapezoid bounded by the lines $y = 1$, the x- and y- axes, and the perpendicular bisector of the segment joining $(0,0)$ and $(3,1)$. The perpendicular bisector goes through the point $(3/2, 1/2)$, which is the center of the square whose vertices are $(1,0)$, $(2,0)$, $(2,1)$, and Hence, the line cuts the square into two quadrilaterals of equal area $1/2$. Thus the area of the trapezoid is $3/2$ and the probability is $3/4$.

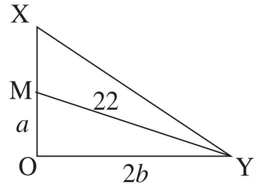


19. **(D)** Adding the given equations gives $2(ab+bc+ca) = 484$, so $ab+bc+ca = 242$. Subtracting from this each of the given equations yields $bc = 90$, $ca = 80$, and $ab = 72$. It follows that $a^2b^2c^2 = 90 \cdot 80 \cdot 72 = 720^2$. Since $abc > 0$, we have $abc = 720$.

20. **(B)** Let $OM = a$ and $ON = b$. Then

$$19^2 = (2a)^2 + b^2 \quad \text{and} \quad 22^2 = a^2 + (2b)^2.$$





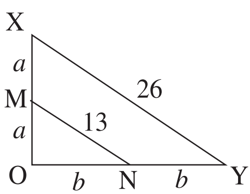
Hence

$$5(a^2 + b^2) = 19^2 + 22^2 = 845.$$

It follows that

$$MN = \sqrt{a^2 + b^2} = \sqrt{169} = 13.$$

Since $\triangle XOY$ is similar to $\triangle MON$ and $XO = 2 \cdot MO$, we have $XY = 2 \cdot MN = 26$.



21. **(A)** Since $2002 = 11 \cdot 13 \cdot 14$, we have

$$a_n = \begin{cases} 11, & \text{if } n = 13 \cdot 14 \cdot i, \text{ where } i = 1, 2, \ldots, 10; \\ 13, & \text{if } n = 14 \cdot 11 \cdot j, \text{ where } j = 1, 2, \ldots, 12; \\ 14, & \text{if } n = 11 \cdot 13 \cdot k, \text{ where } k = 1, 2, \ldots, 13; \\ 0, & \text{otherwise.} \end{cases}$$

Hence $\displaystyle\sum_{n=1}^{2001} a_n = 11 \cdot 10 + 13 \cdot 12 + 14 \cdot 13 = 448.$

22. **(B)** We have $a_n = \frac{1}{\log_n 2002} = \log_{2002} n$, so

$$\begin{aligned} b - c &= (\log_{2002} 2 + \log_{2002} 3 + \log_{2002} 4 + \log_{2002} 5) \\ &\quad - (\log_{2002} 10 + \log_{2002} 11 + \log_{2002} 12 + \log_{2002} 13 + \log_{2002} 14) \\ &= \log_{2002} \frac{2 \cdot 3 \cdot 4 \cdot 5}{10 \cdot 11 \cdot 12 \cdot 13 \cdot 14} = \log_{2002} \frac{1}{11 \cdot 13 \cdot 14} = \log_{2002} \frac{1}{2002} = -1. \end{aligned}$$

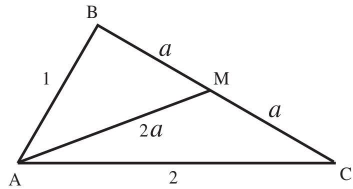
23. **(C)** Let M be the midpoint of $\overline{BC}$, let $AM = 2a$, and let $\theta = \angle AMB$. Then $\cos\angle AMC = -\cos\theta$. Applying the Law of Cosines to $\triangle ABM$ and to $\triangle AMC$ yields, respectively,

$$a^2 + 4a^2 - 4a^2\cos\theta = 1$$

and

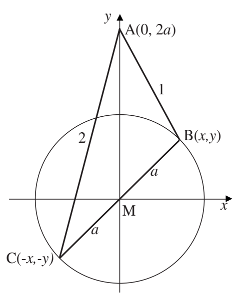
$$a^2 + 4a^2 + 4a^2\cos\theta = 4.$$

Adding, we obtain $10a^2 = 5$, so $a = \sqrt{2}/2$ and $BC = 2a = \sqrt{2}$.



OR

As above, let M be the midpoint of $\overline{BC}$ and $AM = 2a$. Put a rectangular coordinate system in the plane of the triangle with the origin at M so that A has coordinates $(0, 2a)$. If the coordinates of B are (x, y), then the point C has coordinates $(-x, -y)$,



so

$$x^2 + (2a - y)^2 = 1 \quad \text{and} \quad x^2 + (2a + y)^2 = 4.$$

Combining the last two equations gives $2\left(x^2 + y^2\right) + 8a^2 = 5$. But, $x^2 + y^2 = a^2$, so $10a^2 = 5$. Thus, $a = \sqrt{2}/2$ and $BC = \sqrt{2}$.

24. **(E)** We have

$$\text{Area }(ABCD) \le \frac{1}{2} AC \cdot BD,$$

with equality if and only if $AC \perp BD$. Since

$$2002 = \text{Area}(ABCD) \le \frac{1}{2} AC \cdot BD$$
$$\le \frac{1}{2}(AP + PC) \cdot (BP + PD) = \frac{52 \cdot 77}{2} = 2002,$$

it follows that the diagonals AC and BD are perpendicular and intersect at P. Thus, $AB = \sqrt{24^2 + 32^2} = 40$, $BC = \sqrt{28^2 + 32^2} = 4\sqrt{113}$, $CD = \sqrt{28^2 + 45^2} = 53$, and $DA = \sqrt{45^2 + 24^2} = 51$. The perimeter of $ABCD$ is therefore

$$144 + 4\sqrt{113} = 4\left(36 + \sqrt{113}\right).$$

25. **(E)** Note that

$$f(x) + f(y) = x^2 + 6x + y^2 + 6y + 2 = (x+3)^2 + (y+3)^2 - 16$$

and

$$f(x) - f(y) = x^2 - y^2 + 6(x - y) = (x - y)(x + y + 6).$$

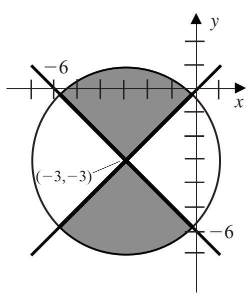
The given conditions can be written as

$$(x+3)^2 + (y+3)^2 \le 16 \quad \text{and} \quad (x - y)(x + y + 6) \le 0.$$

The first inequality describes the region on and inside the circle of radius 4 with center $(-3, -3)$. The second inequality can be rewritten as

$$(x - y \ge 0 \text{ and } x + y + 6 \le 0) \quad \text{or} \quad (x - y \le 0 \text{ and } x + y + 6 \ge 0).$$

Each of these inequalities describes a half-plane bounded by a line that passes through $(-3, -3)$ and has slope 1 or -1. Thus, the set R is the shaded region in the following diagram, and its area is half the area of the circle, which is $8\pi \approx 25.13$.



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