1. **(D)** It is given that $0.1x = 2$ and $0.2y = 2$, so $x = 20$ and $y = 10$. Thus $x - y = 10$.

2. **(B)** Since $2x + 7 = 3$ we have $x = -2$. Hence

$$-2 = bx - 10 = -2b - 10, \quad \text{so} \quad 2b = -8, \text{ and } b = -4.$$

3. **(B)** Let w be the width of the rectangle. Then the length is $2w$, and

$$x^2 = w^2 + (2w)^2 = 5w^2.$$

The area is consequently $w(2w) = 2w^2 = \frac{2}{5}x^2$.

4. **(A)** If Dave buys seven windows separately he will purchase six and receive one free, for a cost of \$600. If Doug buys eight windows separately, he will purchase seven and receive one free, for a total cost of \$700. The total cost to Dave and Doug purchasing separately will be \$1300. If they purchase fifteen windows together, they will need to purchase only 12 windows, for a cost of \$1200, and will receive 3 free. This will result in a savings of \$100.

5. **(B)** The sum of the 50 numbers is $20 \cdot 30 + 30 \cdot 20 = 1200$. Their average is $1200/50 = 24$.

6. **(B)** Because (rate)(time) = (distance), the distance Josh rode was $(4/5)(2) = 8/5$ of the distance that Mike rode. Let m be the number of miles that Mike had ridden when they met. Then the number of miles between their houses is

$$13 = m + \frac{8}{5}m = \frac{13}{5}m.$$

Thus $m = 5$.

7. **(C)** The symmetry of the figure implies that $\triangle ABH$, $\triangle BCE$, $\triangle CDF$, and $\triangle DAG$ are congruent right triangles. So

$$BH = CE = \sqrt{BC^2 - BE^2} = \sqrt{50 - 1} = 7,$$

and $EH = BH - BE = 7 - 1 = 6$. Hence the square $EFGH$ has area $6^2 = 36$.

OR

As in the first solution, $BH = 7$. Now note that $\triangle ABH$, $\triangle BCE$, $\triangle CDF$, and $\triangle DAG$ are congruent right triangles, so

$$\text{Area}(EFGH) = \text{Area}(ABCD) - 4\text{Area}(\triangle ABH) = 50 - 4\left(\tfrac{1}{2} \cdot 1 \cdot 7\right) = 36.$$

8. **(D)** Since A, M, and C are digits we have

$$0 \le A + M + C \le 9 + 9 + 9 = 27.$$

The prime factorization of 2005 is $2005 = 5 \cdot 401$, so

$$100A + 10M + C = 401 \quad \text{and} \quad A + M + C = 5.$$

Hence $A = 4$, $M = 0$, and $C = 1$.

9. **(A)** The quadratic formula yields

$$x = \frac{-(a+8) \pm \sqrt{(a+8)^2 - 4\cdot 4\cdot 9}}{2\cdot 4}.$$

The equation has only one solution precisely when the value of the discriminant, $(a+8)^2 - 144$, is 0. This implies that $a = -20$ or $a = 4$, and the sum is -16.

OR

The equation has one solution if and only if the polynomial is the square of a binomial with linear term $\pm\sqrt{4x^2} = \pm 2x$ and constant term $\pm\sqrt{9} = \pm 3$. Because $(2x \pm 3)^2$ has a linear term $\pm 12x$, it follows that $a + 8 = \pm 12$. Thus a is either -20 or 4, and the sum of those values is -16.

10. **(B)** The unit cubes have a total of $6n^3$ faces, of which $6n^2$ are red. Therefore

$$\frac{1}{4} = \frac{6n^2}{6n^3} = \frac{1}{n}, \quad \text{so} \quad n = 4.$$

11. **(E)** The first and last digits must be both odd or both even for their average to be an integer. There are $5\cdot 5 = 25$ odd-odd combinations for the first and last digits. There are $4\cdot 5 = 20$ even-even combinations that do not use zero as the first digit. Hence the total is 45.

12. **(D)** The slope of the line is

$$\frac{1000-1}{100-1} = \frac{111}{11},$$

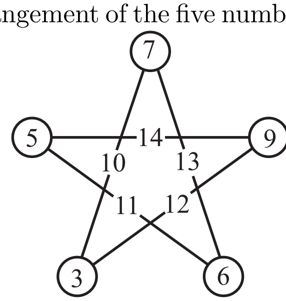
so all points on the line have the form $(1+11t, 1+111t)$. Such a point has integer coordinates if and only if t is an integer, and the point is strictly between A and B if and only if $0 < t < 9$. Thus there are 8 points with the required property.

13. **(D)** Each number appears in two sums, so the sum of the sequence is

$$2(3+5+6+7+9) = 60.$$

The middle term of a five-term arithmetic sequence is the mean of its terms, so $60/5 = 12$ is the middle term.

The figure shows an arrangement of the five numbers that meets the requirement.



14. **(D)** A standard die has a total of 21 dots. For $1 \le n \le 6$, a dot is removed from the face with n dots with probability $n/21$. Thus the face that originally has n dots is left with an odd number of dots with probability $n/21$ if n is even and $1 - n/21$ if n is odd. Each face is the top face with probability $1/6$. Therefore the top face has an odd number of dots with probability

$$\begin{aligned}\frac{1}{6}\left(\left(1-\frac{1}{21}\right)+\frac{2}{21}+\left(1-\frac{3}{21}\right)+\frac{4}{21}+\left(1-\frac{5}{21}\right)+\frac{6}{21}\right) &= \frac{1}{6}\left(3+\frac{3}{21}\right)\\ &= \frac{1}{6}\cdot\frac{66}{21}=\frac{11}{21}.\end{aligned}$$

OR

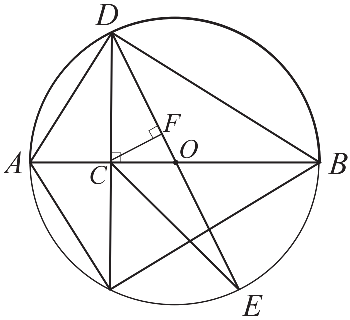
The probability that the top face is odd is $1/3$ if a dot is removed from an odd face, and the probability that the top face is odd is $2/3$ if a dot is removed from an even face. Because each dot has the probability $1/21$ of being removed, the top face is odd with probability

$$\left(\frac{1}{3}\right)\left(\frac{1+3+5}{21}\right)+\left(\frac{2}{3}\right)\left(\frac{2+4+6}{21}\right)=\frac{33}{63}=\frac{11}{21}.$$

15. **(C)** Let O be the center of the circle. Each of $\triangle DCE$ and $\triangle ABD$ has a diameter of the circle as a side. Thus the ratio of their areas is the ratio of the two altitudes to the diameters. These altitudes are $\overline{DC}$ and the altitude from C to $\overline{DO}$ in $\triangle DCE$. Let F be the foot of this second altitude. Since $\triangle CFO$ is similar to $\triangle DCO$,

$$\frac{CF}{DC}=\frac{CO}{DO}=\frac{AO-AC}{DO}=\frac{\frac{1}{2}AB-\frac{1}{3}AB}{\frac{1}{2}AB}=\frac{1}{3},$$

which is the desired ratio.



OR

Because $AC = AB/3$ and $AO = AB/2$, we have $CO = AB/6$. Triangles DCO and DAB have a common altitude to $\overline{AB}$ so the area of $\triangle DCO$ is $\frac{1}{6}$ the area of $\triangle ADB$. Triangles DCO and ECO have equal areas since they have a common base $\overline{CO}$ and their altitudes are equal. Thus the ratio of the area of $\triangle DCE$ to the area of $\triangle ABD$ is $1/3$.

16. **(D)** Consider a right triangle as shown. By the Pythagorean Theorem,

$$(r+s)^2 = (r-3s)^2 + (r-s)^2$$

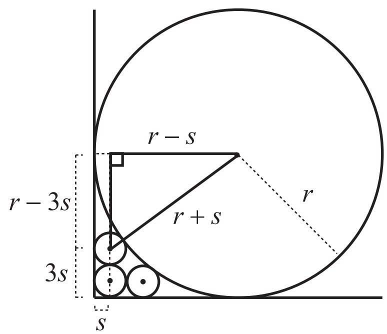
so

$$r^2 + 2rs + s^2 = r^2 - 6rs + 9s^2 + r^2 - 2rs + s^2$$

and

$$0 = r^2 - 10rs + 9s^2 = (r-9s)(r-s).$$

But $r \neq s$, so $r = 9s$ and $r/s = 9$.

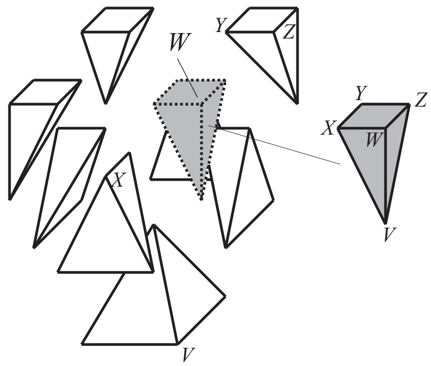


OR

Because the ratio r/s is independent of the value of s, assume that $s = 1$ and proceed as in the previous solution.

17. **(A)** The piece that contains W is shown. It is a pyramid with vertices V, W, X, Y, and Z. Its base $WXYZ$ is a square with sides of length $1/2$ and its altitude VW is 1. Hence the volume of this pyramid is

$$\frac{1}{3}\left(\frac{1}{2}\right)^2(1) = \frac{1}{12}.$$



18. **(A)** Of the numbers less than 1000, 499 of them are divisible by two, 333 are divisible by 3, and 199 are divisible by 5. There are 166 multiples of 6, 99 multiples of 10, and 66 multiples of 15. And there are 33 numbers that are divisible by 30. So by the Inclusion-Exclusion Principle there are

$$499 + 333 + 199 - 166 - 99 - 66 + 33 = 733$$

numbers that are divisible by at least one of 2, 3, or 5. Of the remaining $999 - 733 = 266$ numbers, 165 are primes other than 2, 3, or 5. Note that 1 is neither prime nor composite. This leaves exactly 100 prime-looking numbers.

19. **(B)** Because the odometer uses only 9 digits, it records mileage in base-9 numerals, except that its digits 5, 6, 7, 8, and 9 represent the base-9 digits 4, 5, 6, 7, and 8. Therefore the mileage is

$$2004_{\text{base } 9} = 2 \cdot 9^3 + 4 = 2 \cdot 729 + 4 = 1462.$$

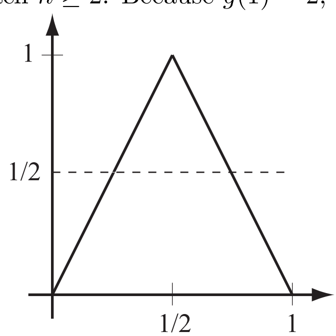
OR

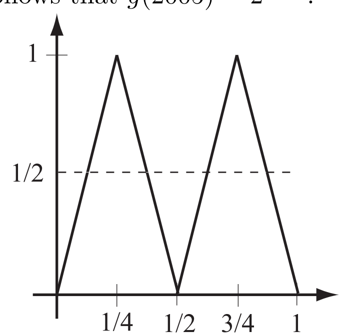
The number of miles traveled is the same as the number of integers between 1 and 2005, inclusive, that do not contain the digit 4. First consider the integers less than 2000. There are two choices for the first digit, including 0, and 9 choices for each of the other three. Because one combination of choices is 0000, there are $2 \cdot 9^3 - 1 = 1457$ positive integers less than 2000 that do not contain the digit 4. There are 5 integers between 2000 and 2005, inclusive, that do not have a 4 as a digit, so the car traveled $1457 + 5 = 1462$ miles.

20. **(E)** The graphs of $y = f(x)$ and $y = f^{[2]}(x)$ are shown below. For $n \geq 2$ we have

$$f^{[n]}(x) = f^{[n-1]}(f(x)) = \begin{cases} f^{[n-1]}(2x), & \text{if } 0 \leq x \leq \frac{1}{2}, \\ f^{[n-1]}(2-2x), & \text{if } \frac{1}{2} \leq x \leq 1. \end{cases}$$

Let $g(n)$ be the number of values of x in $[0, 1]$ for which $f^{[n]}(x) = 1/2$. Then $f^{[n]}(x) = 1/2$ for $g(n-1)$ values of x in $[0, 1/2]$ and $g(n-1)$ values of x in $[1/2, 1]$. Furthermore $f^{[n]}(1/2) = f^{[n-1]}(1) = 0 \neq 1/2$ for $n \geq 2$. Hence $g(n) = 2g(n-1)$ for each $n \geq 2$. Because $g(1) = 2$, it follows that $g(2005) = 2^{2005}$.





21. **(C)** The two equations are equivalent to $b = a^{(c^{2005})}$ and $c = 2005 - b - a$, so

$$c = 2005 - a^{(c^{2005})} - a.$$

If $c > 1$, then

$$b \geq 2^{(2^{2005})} > 2005 > 2005 - a - c = b,$$

which is a contradiction. For $c = 0$ and for $c = 1$, the only solutions are the ordered triples $(2004, 1, 0)$ and $(1002, 1002, 1)$, respectively. Thus the number of solutions is 2.

22. **(B)** Let the dimensions of P be x, y, and z. The sum of the lengths of the edges of P is $4(x + y + z)$, and the surface area of P is $2xy + 2yz + 2xz$, so

$$x + y + z = 28 \quad \text{and} \quad 2xy + 2yz + 2xz = 384.$$

Each internal diagonal of P is a diameter of the sphere, so

$$(2r)^2 = (x^2 + y^2 + z^2) = (x + y + z)^2 - (2xy + 2xz + 2yz) = 28^2 - 384 = 400.$$

So $2r = 20$ and $r = 10$.

Note: There are infinitely many positive solutions of the system $x+y+z = 28$, $2xy+2yz+2xz = 384$, so there are infinitely many non-congruent boxes meeting the given conditions, but each can be inscribed in a sphere of radius 10.

23. **(B)** Let $a = 2^j$ and $b = 2^k$. Then

$$\log_a b = \log_{2^j} 2^k = \frac{\log 2^k}{\log 2^j} = \frac{k \log 2}{j \log 2} = \frac{k}{j},$$

so $\log_a b$ is an integer if and only if k is an integer multiple of j. For each j, the number of integer multiples of j that are at most 25 is $\left\lfloor \frac{25}{j} \right\rfloor$. Because $j \neq k$, the number of possible values of k for each j is $\left\lfloor \frac{25}{j} \right\rfloor - 1$. Hence the total number of ordered pairs (a, b) is

$$\sum_{j=1}^{25} \left(\left\lfloor \frac{25}{j} \right\rfloor - 1 \right) = 24 + 11 + 7 + 5 + 4 + 3 + 2(2) + 4(1) = 62.$$

Since the total number of possibilities for a and b is $25 \cdot 24$, the probability that $\log_a b$ is an integer is

$$\frac{62}{25 \cdot 24} = \frac{31}{300}.$$

24. **(B)** The polynomial $P(x) \cdot R(x)$ has degree 6, so $Q(x)$ must have degree 2. Therefore Q is uniquely determined by the ordered triple $(Q(1), Q(2), Q(3))$. When $x = 1$, 2, or 3, we have $0 = P(x) \cdot R(x) = P(Q(x))$. It follows that $(Q(1), Q(2), Q(3))$ is one of the 27 ordered triples (i, j, k), where i, j, and k can

be chosen from the set $\{1, 2, 3\}$. However, the choices $(1, 1, 1)$, $(2, 2, 2)$, $(3, 3, 3)$, $(1, 2, 3)$, and $(3, 2, 1)$ lead to polynomials $Q(x)$ defined by $Q(x) = 1$, 2, 3, x, and $4 - x$, respectively, all of which have degree less than 2. The other 22 choices for $(Q(1), Q(2), Q(3))$ yield non-collinear points, so in each case $Q(x)$ is a quadratic polynomial.

25. **(C)** Let $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, and $C(x_3, y_3, z_3)$ be the vertices of such a triangle. Let

$$(\Delta x_k, \Delta y_k, \Delta z_k) = (x_{k+1} - x_k, y_{k+1} - y_k, z_{k+1} - z_k), \text{ for } 1 \le k \le 3,$$

where $(x_4, y_4, z_4) = (x_1, y_1, z_1)$. Then $(|\Delta x_k|, |\Delta y_k|, |\Delta z_k|)$ is a permutation of one of the ordered triples $(0, 0, 1)$, $(0, 0, 2)$, $(0, 1, 1)$, $(0, 1, 2)$, $(0, 2, 2)$, $(1, 1, 1)$, $(1, 1, 2)$, $(1, 2, 2)$, or $(2, 2, 2)$. Since $\triangle ABC$ is equilateral, $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ correspond to permutations of the same ordered triple (a, b, c). Because

$$\sum_{k=1}^{3} \Delta x_k = \sum_{k=1}^{3} \Delta y_k = \sum_{k=1}^{3} \Delta z_k = 0,$$

the sums

$$\sum_{k=1}^{3} |\Delta x_k|, \quad \sum_{k=1}^{3} |\Delta y_k|, \quad \text{and} \quad \sum_{k=1}^{3} |\Delta z_k|$$

are all even. Therefore $(|\Delta x_k|, |\Delta y_k|, |\Delta z_k|)$ is a permutation of one of the triples $(0, 0, 2)$, $(0, 1, 1)$, $(0, 2, 2)$, $(1, 1, 2)$, or $(2, 2, 2)$.

If $(a, b, c) = (0, 0, 2)$, each side of $\triangle ABC$ is parallel to one of the coordinate axes, which is impossible.

If $(a, b, c) = (2, 2, 2)$, each side of $\triangle ABC$ is an interior diagonal of the $2 \times 2 \times 2$ cube that contains S, which is also impossible.

If $(a, b, c) = (0, 2, 2)$, each side of $\triangle ABC$ is a face diagonal of the $2 \times 2 \times 2$ cube that contains S. The three faces that join at any vertex determine such a triangle, so the triple $(0, 2, 2)$ produces a total of 8 triangles.

If $(a, b, c) = (0, 1, 1)$, each side of $\triangle ABC$ is a face diagonal of a unit cube within the larger cube that contains S. There are 8 such unit cubes producing a total of $8 \cdot 8 = 64$ triangles.

There are two types of line segments for which $(a, b, c) = (1, 1, 2)$. One type joins the center of the face of the $2 \times 2 \times 2$ cube to a vertex on the opposite face. The other type joins the midpoint of one edge of the cube to the midpoint of another edge. Only the second type of segment can be a side of $\triangle ABC$. The midpoint of each of the 12 edges is a vertex of two suitable triangles, so there are $12 \cdot 2/3 = 8$ such triangles.

The total number of triangles is $8 + 64 + 8 = 80$.